NASA Technical Memorandum 113087

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October 1997

National Aeronautics and
Space Administration

NOTICE

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ABSTRACT

The newly developed Yeh-Stratton (Y-S) Strength Criterion was used to study the failure of composite materials with central holes and normal cracks. To evaluate the interaction parameters for the Y-S failure theory, it is necessary to perform several biaxial loading tests. However, it is indisputable that the inhomogeneous and anisotropic nature of composite materials have made their own contribution to the complication of the biaxial testing problem. To avoid the difficulties of performing many biaxial tests and still consider the effects of the interaction term in the Y-S Criterion, a simple modification of the Y-S Criterion was developed. The preliminary predictions by the modified Y-S Criterion were relatively conservative compared to the testing data. Thus, the modified Y-S Criterion could be used as a design tool. To further understand the composite failure problem, an investigation of the damage zone in front of the crack tip coupled with the Y-S Criterion is imperative.

INTRODUCTION

Composites have found many applications as advanced engineering materials, effectively employed in various structural systems such as aircraft, automobiles, and power plants. The safety and reliability of these systems are dependent on the design of the constituent components. These components are often subjected to complex service loading conditions, in which two or three dynamic or static principal stresses may exist.

To utilize these advanced materials to their full potential, the establishment of the strength criterion is important. Considerable efforts have been devoted to recent developments of strength/failure criteria for anisotropic materials [1]. Some of the currently existing anisotropic strength criteria were only extensions of isotropic yield criteria. The newly developed Y-S Failure Criterion for composite materials goes beyond by incorporating yield/failure functions for isotropic materials [2].

THE YEH-STRATTON FAILURE CRITERION

The generalized but simplified form of the Y-S failure function, *f*, for composite materials can be written as below,

$$f = A_i \sigma_i + A_j \sigma_j + B_{ij} \sigma_i \sigma_j + C_{ij} T^2_{ij} = 1 \qquad (1)$$

for i, j = 1, 2, 3, where A_i, A_j, B_{ij} and C_{ij} are constants determined by experiment. The repeated index is not summed and the function f is assumed to be unaffected by changing the sign of the shear stress.

At a glance, equation (1) is of the Tsai-Wu type failure criterion. However, there are certain differences between the generalized Y-S and the Tsai-Wu Criterion. The Tsai-Wu yield surface is defined by the single yield function of the form $F_i \sigma_i + F_{ij} \sigma_i \sigma_j = 1$ (i, j = 1, 2,...6) where F_i and F_{ij} are strength tensors of the second and fourth rank, respectively. The shape of the yield surface which is subjected to the stability conditions will be an ellipsoid. However, the generalized Y-S Criterion does not define the entire failure surface by a single equation. Rather, the entire failure surface must consist of piecewise surfaces. Each surface can be formed by a similar failure function shown in equation (1) for each quadrant of the stress space, and thus, all the constants in equation (1) must be evaluated by experimental values from each quadrant. Even though each surface may be defined to be hyperboloid, as seen from equation (1), the yield surfaces must be closed.

The concept of a piecewise failure function can be useful for anisotropic materials because anisotropic materials show different strengths in different directions or planes. If the stress states are compressive, the yield function must be adjusted to accommodate the compressive strength in the function as a dominant critical strength, and not tensile strength. It is true that for anisotropic materials, the number of failure functions can equal the number of planes with different material strengths. For isotropic materials, only one failure function can be used for every possible failure stress state and for every plane. By definition of isotropy, the properties of materials are not directional dependent. The constants A_i, A_j and C_{ij} in equation (1) can be computed rather easily if the simple tension, compression and torsion tests are performed while the coefficient B_{ij} of the interaction term is evaluated with biaxial experiments.

THE INTERACTION TERM AND BIAXIAL TESTS

A considerable amount of work concerning the behavior of engineering materials under multiaxial stress/strain conditions has been undertaken on metallic materials for several decades [3]. Most investigations are carried out with the help of simplifying the complex multiaxial loading into various combinations of biaxial loading conditions, using tubular and cruciform plate specimens. The number of similar investigations on composite materials is considerably smaller, although some of the above aspects have been studied [4]. It is indisputable that the inhomogeneous and anisotropic nature of composite materials have made their own contribution to the complication of the problem.

The effects of interaction component B_{ij}, in the generalized Y-S Failure Criterion was studied [5]. The results indicated that for the off-axis strength test, it was recommended that the generalized Y-S Failure Criterion be calibrated with the experimental result of the off-axis strength test with the fiber orientation of $\theta = 12$ degrees. The biaxial strength test, as a result of the conditions on the interaction components, revealed that the failure surface of fiber reinforced composites must lie inside the failure surface defined by the maximum stress theory of anisotropic materials. The interaction components from each quadrant must be calibrated with the experimental data from the corresponding quadrant. For the boron-epoxy (AVCO 5505) with the off-axis test, the reasonable agreement between the Y-S Criterion and the experimental data can be shown without considering the interaction term B_{ij} . For the biaxial strength test, caution must be exercised if the interaction term is ignored.

To avoid the difficulties of running many biaxial/multiaxial tests and to still consider the effects of the interaction term in the Y-S Failure Criterion, a simple modification of the Y-S Criterion was developed as follows: Since the strength of a lamina should depend on the lamina angle and be symmetric with respect to the angle, the strength function of an off-axis lamina may be expanded in the trigonometric Fourier series. The interaction term can be expressed as:

$$B_{12} = [C_o + C_2\cos 2\varphi + C_4\cos 4\varphi]\,[C_o + C_2\cos 2(\pi/2 - \varphi) + C_4\cos 4(\pi/2 - \varphi)] \qquad (2)$$

where φ is the fiber orientation angle and C_o , C_2 and C_4 are functions of the tensile/compressive strength of single ply under different orientation angles [6].

RESULTS AND CONCLUSIONS

To investigate this modified Y-S Criterion, the experimental results obtained by Tan [7] for AS4/3502 $[\pm\theta_2]_s$ family of laminates with central holes and normal cracks were examined. The results predicted by the modified Y-S Criterion are listed in tables 1 and 2.

Table 1. Comparison between the modified Y-S predictions and test results of AS4/3502 $[\pm\theta_2]_s$ laminates with normal cracks.

Crack angle, deg	Crack length a, in.	Damage zone R, in.	Experimental results, ksi	Predicted results, ksi
30°				
	0.68	0.02	52.90	38.78
	1.34	0.03	47.70	35.59
	3.74	0.07	42.30	32.36
	5.02	0.08	39.40	30.72
	7.58	0.14	41.30	31.80
45°				
	0.74	0.10	116.50	104.40
	1.37	0.14	101.40	90.70
	3.77	0.32	95.80	84.40
	5.03	0.40	91.00	81.18
	7.58	0.64	93.80	83.65
60°				
	0.65	0.74	349.60	268.40
	1.30	1.32	327.50	252.30
	3.81	4.16	339.90	261.20
	5.07	5.76	347.50	266.40
	7.57	9.60	368.20	281.50

Table 2. Comparison between the modified Y-S predictions and test results of AS4/3502 $[\pm\theta_2]_S$ laminates with central holes.

Hole	Hole radius a, in.	Damage zone R, in.	Experimental results, ksi	Predicted results, ksi
30°				
	0.59	0.02	51.40	38.27
	1.26	0.03	45.90	34.37
	3.80	0.06	41.20	30.05
	5.18	0.07	37.70	28.00
	7.76	0.11	38.80	28.54
45°				
	0.59	0.09	121.40	111.70
	2.59	0.26	102.20	91.90
	3.83	0.32	93.30	83.30
	5.18	0.37	86.50	76.70
	7.72	0.58	89.40	79.20
60°				
	0.74	0.59	364.90	283.40
	1.40	1.26	343.40	263.50
	2.64	3.81	268.10	208.10
	3.28	5.17	258.80	199.40
	5.60	7.74	277.80	212.70

In this study, the modified Y-S Criterion provided conservative strength predictions for AS4/3502 $[\pm\theta_2]_S$ composite laminates with center holes or normal cracks. Thus, the modified Y-S Failure Criterion could be used for composite laminate design. Since the experimental results listed in tables 1 and 2 are for tensile loadings only, to further evaluate the modified Y-S Criterion, the biaxial or multiaxial testing results are definitely required. It is also noted that in some cases the damage zone size is close to or greater than the original crack length. This is a violation of the basic assumption in linear elastic fracture mechanics theory on which the whole analytical stress calculation was based. Thus, this presents a challenging task in the area of fracture research of composite materials, and further research between the modified Y-S Failure Criterion and the size of the damage zone.

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	October 1997	Technical Memorandum

4. TITLE AND SUBTITLE

Failure Study of Composite Materials by the Yeh-Stratton Criterion

5. FUNDING NUMBERS

WU 529-50-14

6. AUTHOR(S)

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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

NASA Dryden Flight Research Center
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Edwards, California 93523-0273

8. PERFORMING ORGANIZATION REPORT NUMBER

H-2201

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

National Aeronautics and Space Administration
Washington, DC 20546-0001

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

NASA TM-113087

11. SUPPLEMENTARY NOTES

Dr. Hsien-Yang Yeh's work was conducted in support of NASA Contract NCC 4-107.

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Unclassified—Unlimited
Subject Category 24

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

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14. SUBJECT TERMS

Composite materials, Composite research, Failure theory, Macromechanics, Stress concentrations

15. NUMBER OF PAGES

9

16. PRICE CODE

A02

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	Unlimited